Review:

Disturbances: Outside influences that act on a robot while it tries to stay on a path

Trajectory: The path the robot is supposed to be following. It is made up of waypoints that have position, and maybe velocity, or time, or other variables associated with each of them.

Proportional Feedback Control: It is a common solution for keeping a robot on a trajectory. This is when you compare the current value to the desired value to determine the error. The output signal is proportional to the size of the error.

We'll use the **vector format** notation for a line and define two in particular:

$$\overrightarrow{line}_{start\ to\ end} = \overrightarrow{line}_{S\to E} = \begin{bmatrix} End_x - Start_x \\ End_y - Start_y \end{bmatrix}$$

$$\overrightarrow{line}_{start\ to\ robot} = \overrightarrow{line}_{S\to R} = \begin{bmatrix} robot_x - start_x \\ robot_y - start_y \end{bmatrix}$$

We can use the **cross product** to determine which side we are on:

$$side = \overrightarrow{line}_{S\to E} \times \overrightarrow{line}_{S\to R} = \begin{bmatrix} 0 \\ 10 \end{bmatrix} \times \begin{bmatrix} 2 \\ 4 \end{bmatrix}$$

$$\begin{bmatrix} A \\ C \end{bmatrix} \times \begin{bmatrix} B \\ D \end{bmatrix} \to AD - BC$$

Negative means we are on the right side, positive on the left side.
We can find our distance from the line using the dot product and the normal vector:

$$distance = \frac{\left|\overrightarrow{line}_{S\to R} \cdot \overrightarrow{normal}\right|}{\left\|\overrightarrow{normal}\right\|}$$

We can find the normal by taking the original line, swapping the x and y coordinates, and negating one of them.

$$\overrightarrow{Line} = \begin{bmatrix} X \\ Y \end{bmatrix} \to \overrightarrow{Normal} = \begin{bmatrix} -Y \\ X \end{bmatrix}$$

For the magnitude, we take the **Pythagorean theorem** to find the total length:

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$$a^2 + b^2 = c^2$$

Where a and b are the x and y components, and c is the total length.

$$\begin{bmatrix} A \\ C \end{bmatrix} \cdot \begin{bmatrix} B \\ D \end{bmatrix} \rightarrow AB + CD$$

For the **dot product**, we multiply the x terms of both arguments, multiply the two y terms, then add the results.

Final Step:
Translate the error signal into a steering angle. We use a constant factor called the proportional gain, to both scale and translate the error into the correct units.

Error × proportional gain = command
E × K = command

K = proportional gain
E = Error

$$K = \frac{\_\_\_\_^\circ}{\text{meter}}$$

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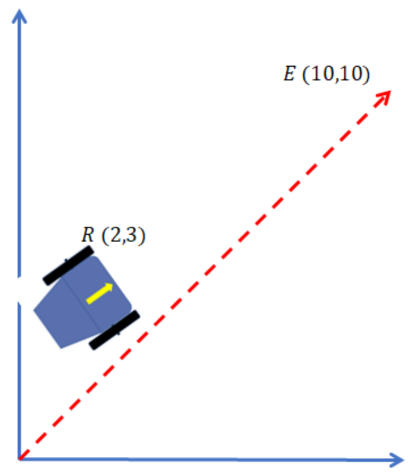
Challenge Questions:

Our robot wants to follow a diagonal line going through the origin and a point at **10,10**. If the robot is at **2,3**, how far away from the line is the robot? And on which side?

We start by expressing the diagonal trajectory as a line vector

$$line_{S-E} = [10 - 0 \quad 10 - 0] = [10 \quad 10]$$



The normal to the trajectory line is

$$normal_{S-E} = [-10 \quad 10]$$

The line from the start of the trajectory to the robot is

$$line_{S-R} = [(2 - 0)\ (3 - 0)] = [2 \quad 3]$$

The distance of the robot to the trajectory is computed using the dot product:

$$distance = \frac{|line_{S-R} \cdot normal_{S-E}|}{||normal_{S-E}||}$$

$$\frac{|[2 \quad 3] \cdot [-10 \quad 10]|}{\sqrt{(-10)^2 + 10^2}} = \frac{|-20+30|}{\sqrt{200}} = \frac{|-10|}{14.142} = 0.707m$$

The orientation of the robot with respect to the trajectory is computed using the cross product

$$side = line_{S-E} \times line_{S-R}$$

$$[10 \quad 10] \times [2 \quad 3] = 10 \cdot 3 - 10 \cdot 2 = +10$$

Since the cross product is positive, the robot is on the *left* of the trajectory

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If the proportional gain is **15** degrees per meter, what is the commanded steering angle from the controller?

From the previous problem we know that the robot is a at distance of 0.707 m to the left of the trajectory

The proportional gain is 15 deg/meter

The magnitude of the steering command can thus be computed as:

$$\begin{aligned} \text{steering angle} &= \text{k} \cdot \text{distance} \\ &= 15 \cdot 0.707 \\ &= 10.605 \text{ degrees} \end{aligned}$$

Since the robot is to the left of the trajectory we want it to steer towards the right (i.e steer clockwise)

Hence, the steering command should be negative.

The final commanded steering angle is.

$$\text{steering angle} = \text{-10.605 degrees}$$

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