## Review:

**LIDAR**: LIght Detection And Ranging, LIDAR produces a list of directions, relative bearings with accompanying distances to the obstacle(s) that the light bounced off of

**ϕ** : Greek letter, refers to the relative bearings from the LIDAR

**Polar Coordinates**: consist of an angle and a range

Robots keep track of position of obstacles on a map

Each cell on the map is either free, an obstacle, or unknown. Some cells we haven't seen yet, others we have a reflection from, and yet others had the sensor beam travel through them without detecting anything.

Since the sensors have some error, we track the probability that each cell is occupied instead of just the binary option of free or occupied.

We use **conditional probability** to keep track of this. We want the probability of their being an obstacle given that there was or was not a return from that cell.

$$p(obstacle|return)$$

However, what we typically have from our sensor specifications is the probability of getting a return if the cell has an obstacle (true positive rate) and the probability of getting a return when the cell is clear (false positive rate).

$$p(return|obstacle) \text{ or } p(return|clear)$$

Since we either get a return or we don't, we know that the probability of not getting a return is one minus the probability of getting a return for each case.

$$p(return|obstacle) = 1 - p(no\ return|obstacle)$$

We can also calculate the probability of getting a return from both cases by taking the probability of each event multiplied by the probability of that event happening.

$$p(return) = p(obstacle) \cdot p(return|obstacle) + p(clear) \cdot p(return|clear)$$

This leads us to **Bayes' Theorem**:

$$p(cell_{new}) = \frac{p(obstacle) \cdot p(cell_{old})}{p(return)}$$

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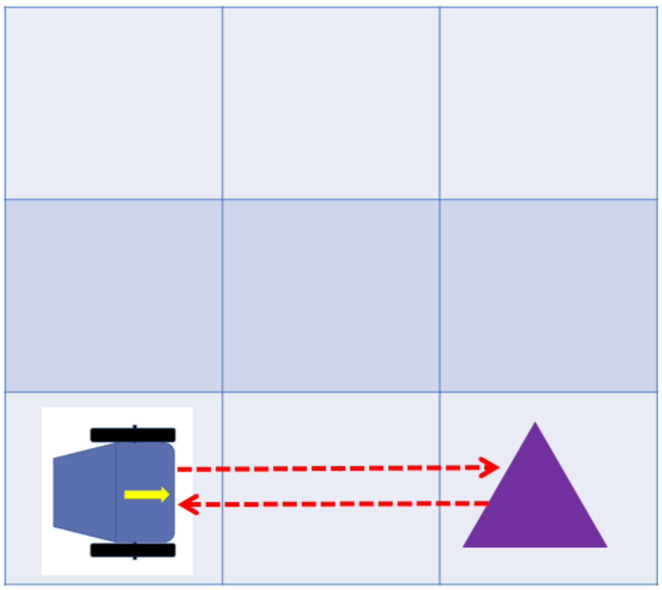
## Challenge Questions:

Start with a 3m by 3m grid with 1m cells as shown. The robot is in the center of the bottom left cell. Initially the map has a value of 50% for all cells. Use:

$p(return|obstacle)$ = 80% and $p(return|clear)$ = 10%



- The robot is in the bottom left cell
- The LIDAR ray travels along the bottom of the grid and is reflected from the bottom right cell.
- Thus, the probability value of bottom right cell needs to be updated since this cell gives us a return measurement.
- In addition, the LIDAR ray passed through the middle cell of the bottom row.
- Hence, the probability value of the middle cell of the bottom row also needs to be updated since it gives us a no-return measurement.

**The robot gets a lidar return from the bottom right cell – Which cells will have a change in value?**

The signal will go through the bottom center cell (no return was detected) and the bottom right cell (return detected. So these two cells will need to be updated.

**What are the updated values for each cell?**

We know the conditional probabilities for a return measurement:

p(return | obstacle) = 80% = 0.8

p(return | clear) = 10% = 0.1

Since there are only two possible measurements (return or no-return), the conditional probability of a no-return measurement when there is an obstacle is:

p(no-return | obstacle) = 1 - 0.8 = 0.2 = 20%

Next we compute the total probability of return and no-return measurements. From the video

p(return) = p(return | obstacle) · p(obstacle) + p(return | clear) · p(clear)

=0.8 · 0.5 + 0.1 · 0.5 = 0.45 = 45%

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Again since there are only two possible measurement values, the total probability of no return is:

p(no-return) = 1 - 0.45 = 0.55 = 55%

Recall for the bottom right cell we got a return measurement. Using Bayes theorem we can update the probability value of this cell:

$$p(cell_{new}) = \frac{p(return \mid obstacle) \cdot p(cell_{old})}{p(return)}$$

= (0.8 · 0.5) / 0.45 = 0.88 = 88%

For the middle cell in the bottom we got a no-return measurement. We can similarly use Bayes theorem to update the probability value of this cell:

$$p(cell_{new}) = \frac{p(no\ return \mid obstacle) \cdot p(cell_{old})}{p(no\ return)}$$

= (0.2 · 0.5) / 0.55 = 0.18 = 18%

If the robot gets a second return from the bottom right cell, what are the updated values?

Once again we can use Bayes theorem to update the probability value of this cell.

However this time the old cell probability value is 88% instead of 50%.

We also have to re-compute the total probability of return using the updated p(obstacle) and p(clear) as:

$$\mathrm{p(return) = p(return \mid obstacle) \cdot p(obstacle) + p(return \mid clear) \cdot p(clear)}$$

$$= 0.8 \cdot 0.88 + 0.1 \cdot 0.12 = 0.716 = 71.6\%$$

$$p(cell_{new}) = \frac{p(return \mid obstacle) \cdot p(cell_{old})}{p(return)}$$

$$= (0.8 \cdot 0.88)\ /\ 0.716 = 0.983 = 98.3\ \%$$

Once again for the middle cell in the bottom row we get a no-return measurement. We can similarly use Bayes theorem to update the probability that these cells contain an obstacle.

We use the old cell probability value as 12% instead of 50% and we re-compute the total probability of no-return:

$$\mathrm{p(no\text{-}return) = 1 - p(return) = 1 - 0.716 = 0.284 = 28.4\%}$$

$$p(cell_{new}) = \frac{p(no\ return \mid obstacle) \cdot p(cell_{old})}{p(no\ return)}$$

$$= (0.2 \cdot 0.12)\ /\ 0.284 = 0.084 = 8.4\%$$

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