Review:

Orientation (heading) is the direction the robot is facing, measured counter-clockwise from the x-axis (typically). This is denoted by theta (θ) and the subscript ‘r’ is for robot (θ_r).

Distance is equal to the amount of rotation of a wheel times the wheel’s radius.

$$S = \theta_r \, r_{wheel}$$

Linear velocity measures the speed in a line:

$$speed = \frac{distance}{time} = linear\ velocity$$

Angular velocity measures the speed of rotation (noted by greek letter omega [ω]):

$$speed\ of\ rotation = \frac{angular\ displacement}{time} = angular\ velocity$$

We can equate linear and angular velocities by dividing our distance equation by time (t)

$$\frac{\theta}{t} = \frac{S \div t}{r}$$

$$\omega = v \div r \quad \text{or} \quad \omega \cdot r = v$$

Using these concepts we can determine our new position based on our starting position (x_0,y_0) and our initial heading θ_0.

$$x_t = x_0 + \omega_{w,avg} \, r_w \, t \cos(\theta_r) \quad t\ 0\ w,avgw\ r$$

$$y_t = y_0 + \omega_{w,avg} \, r_w \, t \sin(\theta_r) \quad t\ 0\ w,avgw\ r$$

$$\omega_{w,avg} = \frac{\omega_{right} + \omega_{left}}{2}$$

$$\theta_{r,t} = \theta_{r,0} + \frac{r_w}{L}(\omega_{right} - \omega_{left})t$$

These equations are only valid for small time steps since θ is changing.

- For slow moving robots 0.1-1 second may be sufficient
- For high speed aircraft 0.1-1 millisecond may be required

Name: ______________________________ Class: _________________

Challenge Questions:

Imagine you have a robot that is **50**cm wide, with **10**cm radius wheels. The robot starts out at (**0**,**0**) with an initial orientation of **π**/4.
If the robot drives both wheels at a constant speed of 1 radian per second for 10 seconds -what is the robot's final position and orientation?

Both wheels are rotating at the same speed so the robot will travel in a straight line
Therefore there is no change in orientation
The total distance traveled will be:

$$distance = \omega_w \cdot r_w \cdot t$$

Also, remember

$$\theta_r = orientation\ of\ the\ robot$$
$$\omega_w = angular\ rate\ of\ the\ wheel$$

The distance can be broken up into two components:

a) **Displacement along the x-axis**

$$\Delta x = distance \cdot cos(\theta) = \omega_w \cdot r_w \cdot t \cdot cos(\theta_r)$$

b) **Displacement along the y-axis**

$$\Delta y = distance \cdot sin(\theta) = \omega_w \cdot r_w \cdot t \cdot sin(\theta_r)$$

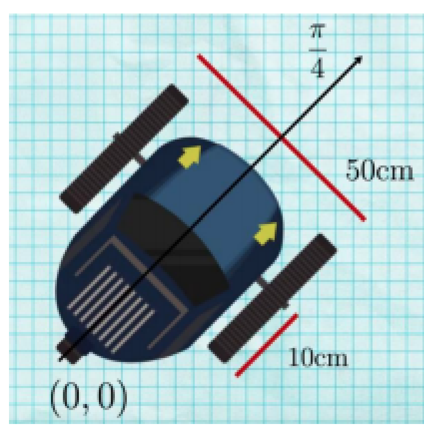
The final position can be calculated as:

$$x_f = x_0 + \Delta x = x_0 + \omega_w \cdot r_w \cdot t \cdot cos(\theta_r)$$
$$y_f = y_0 + \Delta y = y_0 + \omega_w \cdot r_w \cdot t \cdot sin(\theta_r)$$

$$\omega_w = 1\ radian\ per\ second$$
$$r_w = 10cm\ or\ 0.1m$$
$$t = 10\ seconds$$
$$x_0 = 0 \quad y_0 = 0 \quad \theta_r = \frac{\pi}{4}$$

Substituting

$$x_f = 0.707m \quad y_f = 0.707m$$



Name: ______________________________ Class: __________________

Imagine you have a robot that is **50**cm wide, with **10**cm radius wheels. The robot starts out at (**0**,**0**) with an initial orientation of π/4.
The robot runs the right wheel at 1 radian per second and the left wheel at 1.5 radians per second. What is the robot's orientation after 1 sec of motion?

In this question, the wheels are rotating at different speeds so the robot will travel in a circular arc

The distance and final orientation can be computed from:

$$x_t = x_0 + \omega_{w,avg} \cdot r_w \cdot t \cdot cos(\theta_r)$$
$$y_t = y_0 + \omega_{w,avg} \cdot r_w \cdot t \cdot sin(\theta_r)$$
$$\omega_{w,\,avg} = \frac{\omega_{leftwheel} + \omega_{rightwheel}}{2}$$
$$\theta_t = \theta_0 + \omega_r \cdot t$$

The robots angular rate can be calculated from the difference in wheel speeds, the wheel radius, and the distance between the two wheels

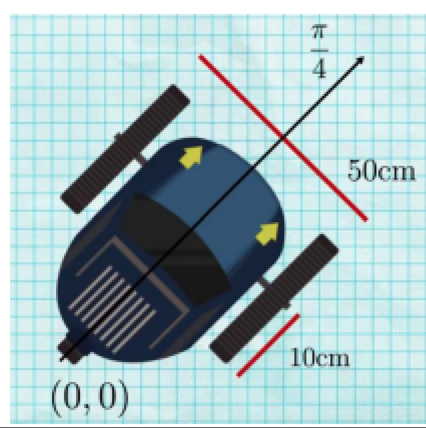
$$\omega_r = \frac{r_w}{L}(\omega_{rightwheel} - \omega_{leftwheel})$$

$$\theta_t = \theta_0 + \frac{r_w}{L}(\omega_{rightwheel} - \omega_{leftwheel})t$$

$$\omega_{leftwheel} = 1.5\ radian\ per\ second$$
$$\omega_{rightwheel} = 1.0\ radian\ per\ second$$
$$r_w = 10cm\ or\ 0.1m$$
$$t = 1\ second$$
$$L = 50cm\ or\ 0.50m$$
$$x_0 = 0 \quad y_0 = 0 \quad \theta_0 = \frac{\pi}{4}$$

Substituting

$$\theta_t = \frac{\pi}{4} + \frac{0.1}{0.5}(1 - 1.5) \cdot 1$$
$$\theta_t = \frac{\pi}{4} - 0.1 \approx 0.685$$



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