Review:

Robots need to find their position on a map relative to landmarks. They must:

1. Identify at least 3 landmarks whose position is known on the map
2. Determine range to the landmark
3. Calculate the intersection point of the range circles

To find the intersection of 3 circles:

1. Use the equation for a circle:
   a. $(x - a)^2 + (y - b)^2 = r^2$
   b. for landmarks located at (a,b) and at a range of r
2. Find the radical line by subtracting the two circle equations
3. Substitute back into one of the circle equations to get a quadratic formula in terms of one variable
4. Solve the quadratic equation to find the two value for that single variable
5. Substitute back into the radical line equation to get the two values for the other variable
6. Substitute these two points into the third circle equation to determine which point the robot is at

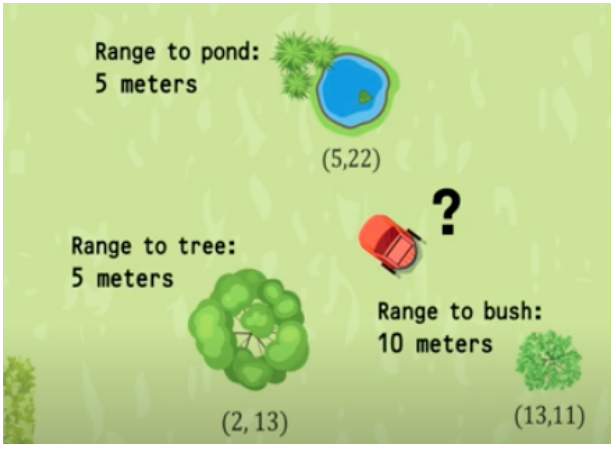
Terms:
Localization = finding its position on a map (the process of getting located)

Formulas:

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

(Quadratic)

Challenge Problem #1:



Challenge Problem #1:

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Challenge Questions:

Your robot is in a yard with a tree, a bush, and a pond. The tree is at (**2**, **13**) and the range is **5** meters. The bush is at (**13**, **11**) and the range is **10** meters. Finally, the pond is at (**5**,**22**) and the range is **5** meters.
What is the robot's position?

$$(range\ to\ object)^2 = (difference\ in\ x)^2 + (difference\ in\ y)^2$$

$$robot\colon x,\ y$$

$$tree\colon (x-2)^2 + (y-13)^2 = 5^2$$

$$bush\colon (x-13)^2 + (y-11)^2 = 10^2$$

$$pond\colon (x-5)^2 + (y-22)^2 = 5^2$$

1) Expand and simplify tree and bush

$$Tree\colon x^2 - 4x + 4 + y^2 - 26y + 169 = 25$$

$$x^2 - 4x + y^2 - 26y + 148 = 0$$

$$bush\colon x^2 - 26x + 169 + y^2 - 22y + 121 = 100$$

$$x^2 - 26x + y^2 - 22y + 190 = 0$$

2) Subtract the two equations

$$-4x + 26x - 26y + 22y + 158 - 190 = 0$$

$$22x - 4y - 42 = 0$$

$$y = \frac{22}{4}x - \frac{42}{4}$$

3) Substitute to find one coordinate (x)

$$Tree\colon x^2 - 4x + \left(\frac{22}{4}x - \frac{42}{4}\right)^2 - 26\left(\frac{22}{4}x - \frac{42}{4}\right) + 148 = 0$$

$$x^2 - 4x + \frac{484}{16}x^2 - \frac{1848}{16}x + \frac{1764}{16} - \frac{572}{4}x + \frac{1092}{4} + 148 = 0$$

$$\frac{500}{16}x^2 - \frac{4200}{16}x + \frac{1092}{16} = 0$$

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$$5x^2 - 42x + 85 = 0$$

4) Solve quadratic

$$x = \frac{42 \pm \sqrt{1764 - 4 \cdot 5 \cdot 85}}{2 \cdot 5}$$

$$x = \frac{42 \pm 8}{10} = 5 \; or \; 3.4$$

5) Find other coordinate (y)

$$y = \frac{22}{4}x - \frac{42}{4}$$

$$y = \frac{22}{4} \cdot 5 - \frac{42}{4} = 17$$

$$y = \frac{22}{4} \cdot 3.4 - \frac{42}{4} = 8.2$$

$$robot\ location\text{: } (5,\ 17)\ or\ (3.4,\ 8.2)$$

6) Try in the "Pond" equation to see which of the two points works

$$(x - 5)^2 + (y - 22)^2 = 5^2$$

$$(5 - 5)^2 + (17 - 22)^2 = 5^2$$

$$0^2 + (-5)^2 = 25$$

$$25 = 25$$

Checks - this could be an answer

$$(x - 5)^2 + (y - 22)^2 = 5^2$$

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$$(3.4 - 5)^2 + (8.2 - 22)^2 = 5^2$$

$$2.56 + 190.44 = 25$$

$$193 \neq 25$$

This one is not true, so the robot cannot be at (3.4, 8.2)

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Now imagine the robot only sees two landmarks, a pile of rocks and an umbrella. The rocks are at the position (**10**,**0**) and the umbrella is at the position (**20**,**0**). The distance to the rocks is **5** meters, and the distance to the umbrella is also **5** meters. Even though there are only two landmarks, show how you can still determine the position of the robot.

1) Expand and simplify

$$rocks: (x - 10)^2 + (y - 0)^2 = 5^2$$

$$x^2 - 20x + 100 + y^2 = 25$$

$$x^2 - 20x + y^2 + 75 = 0$$

$$umbrella: (x - 20)^2 + (y - 0)^2 = 5^2$$

$$x^2 - 40x + 400 + y^2 = 25$$

$$x^2 - 40x + y^2 + 375 = 0$$

2) Subtract

$$20x - 300 = 0$$

$$x = 15$$

3) Substitute to find 2nd coordinate

$$x^2 - 20x + y^2 + 75 = 0$$

$$15^2 - 20 \cdot 15 + y^2 + 75 = 0$$

$$y^2 = 0$$

$$y = \pm 0$$

Two roots at (15,0)

In this case you can because the two circles are tangent at a single point.

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